Parametric Maintenance and Control of Vibration while Deep Hole Drilling

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Abstract. The process of deep hole drilling with intermediate controlled support is considered. The cutting efficiency while the processing of the deep holes is considerably limited due to the low stiffness of the tool. The necessary condition of the process is the formation of thin segmented chip for its removal from the cutting region. One method of chip segmentation is the vibratory drilling when the tool support is excited by the vibrator. The auto-resonant method is also used when the tool axial self-vibrations are excited due to the regeneration mechanism of chip formation. But these methods are inefficient in case of drilling by the long flexible drill bit. The new method of tool excitation in the paper is analyzed when the vibrations are applied to the intermediate support. The model described in the present paper confirms that this method provides: 1) the arising of the tool effective axial stiffness; 2) excitation of the drill bit lateral vibrations due to parametric resonance caused by the periodic variation of the position of the intermediate support.

Keywords:
Drilling, Dynamics, Stability, Vibration.

1 INTRODUCTION

The reliable chip removal from the cutting region is one of the most important problems in deep hole processing. The one way is the applying various vibratory devices exciting the tool support in axial direction [1, 2], or design of special elastic tool fixation which provides realization of auto-resonant regimes of vibratory cutting [3, 4, 5]. Vibration cutting technologies have the special features and applications. In particular, the ensuring of the required cutting conditions requires the deep understanding of the process physics and ability of the complex dynamical phenomenon controlling. The optimal solution selection relates on the elaboration of new schemes and conceptions. It is well known that the parametric vibration excitation may be especially effective in sense of the energy consumption.

The schematic of deep hole drilling by long flexible tool in Fig.1 is presented. The axial vibrations of the tool cutting edges are obtained due to the axial excitation of the intermediate support. The support position $a(t)$ varies periodically in time. The mathematical model of the flexible beam (drill bit) lateral vibrations in case of the intermediate support specified motion in this paper is analyzed. The algorithm of such system analysis is elaborated by the reduction of the non-stationary boundary dynamical problem to the integral equation solution. The vibration analysis in the region of the dynamic instability caused by the parametric loading is possible only regarding the nonlinearities inherent to such systems. The applied method of problem reduction to the finite degree of freedom dynamical system occurs to be very efficient and allows estimating the system as operable as it provides the required axial vibration of cutting edges with the specified frequency and amplitude.

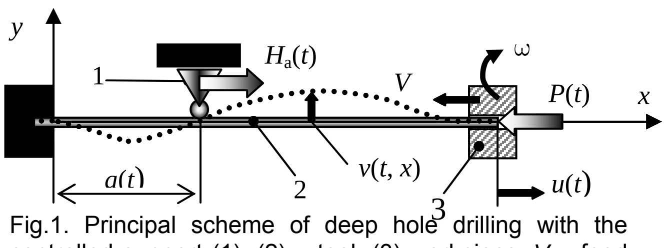

Fig.1. Principal scheme of deep hole drilling with the controlled support (1), (2) – tool, (3) workpiece, V – feed rate, ω - angular speed of workpiece rotation

2 MODEL DESCRIPTION

2.1 Basic relations.

We consider the lateral vibrations of the homogeneous beam in plane (x, y) (Fig.1). Beam has the initial imperfects which are described by the following relations

$$v_0(x), \quad x\in[0,l]; \quad x=0: \quad v_0=0, \\ \frac{d v_0}{d x}=0; \quad x=l: \quad v_0=0, \frac{d v_0}{d x}=0 \tag{1}$$

The total beam axial line deflection $v(t, x)$ from the x axis is presented as the sum

$$v(t,x)=v_0(x)+\Delta v(t,x) \tag{2}$$

The cross section rotation angles with respect the straight configuration are regarded as small: $\vartheta(t,x)=\partial v(t,x)/\partial x, \quad |\vartheta|\ll 1$. The beam (drill bit) is subjected to the axial component of cutting force P. The intermediate support causes the lateral reaction R_a. The support vibrates under action of the horizontal control force H_a. The constraint between the support and the beam is considered as holonomic (ideal), i.e. force R_a is normally directed to the beam axial line (Fig.2).

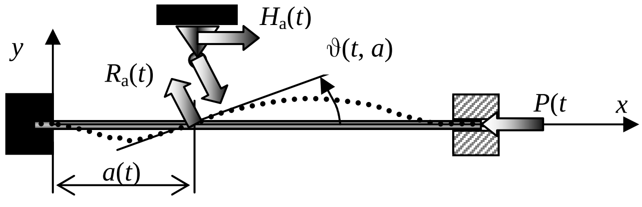

Fig. 2. The schematic diagram of the beam and the intermediate support interaction

If neglect the support inertia the horizontal force acting on the support is determined by the following relation

$$H_a(t)=-R_a(t)\vartheta(t,a) \tag{3}$$

The equations of tool small vibrations are

$$m\frac{\partial^2 \Delta v}{\partial t^2}+EI\frac{\partial^4 \Delta v}{\partial x^4}+P(t)\frac{\partial^2 \Delta v}{\partial x^2}=$$
$$R_a\,\delta\big(x-a(t)\big)-P(t)\frac{\partial^2 v_0}{\partial x^2} \qquad (4)$$

where m is mass per unit length, EI is the beam flexural rigidity with respect z axis (E is Young modulus, I is the cross section inertia moment). The boundary conditions for the function $\Delta v(t,x),\quad x\in[0,l]$ with account of the intermediate support are

$$x=0:\quad \Delta v=0,\frac{\partial \Delta v}{\partial x}=0;$$
$$x=l:\quad \Delta v=0,\frac{\partial \Delta v}{\partial x}=0; \qquad (5)$$
$$x=a:\quad \Delta v+v_0=0$$

The axial displacement of the beam right end $u(t)$ caused by the beam bending, if suppose that the beam is inextensible, are determined as follows

$$u(t)=-\frac{1}{2}\int_0^l\left[\frac{\partial v(t,x)}{\partial x}\right]^2 dx \qquad (6)$$

2.2 Equations Normalization

The scales of time, axial coordinate and displacement T_*, X_*, Y_* are:

$$t=T_*\,\bar t,\quad x=X_*\,\bar x,\Delta v=Y_*\,\xi, v_0=Y_*\,\xi_0,\Delta u=Y_*\,\eta$$
$$T_*=l^2\sqrt{\frac{m}{EI}},X_*=l;Y_*=\varepsilon l,\dot{(\ )}=\frac{\partial(\ )}{\partial \bar t},(\ )'=\frac{\partial(\ )}{\partial \bar x} \qquad (7)$$

The parameter ε accounts that the ratio of the lateral beam deflection to its length is small. For real tools the parameter ε has order of 10^{-2} or less. The scale for forces is chosen as $P_*=EI/l^2$. The normalized variables and parameters we designate by the same but superscripted letters:

$$P(t)=\bar P(\bar t)P_*, R_a=\bar R_a P_*, H_a=\bar H_a P_*, a(t)=\bar a(\bar t)l \qquad (8)$$

By dividing eq. (4) on EIY_*/l^4 and using the introduced notation (7), (8) we arrive at:

$$\ddot\xi+\xi''''+P(t)\xi''=\frac{1}{\varepsilon}R_a\,\delta\big(x-a(t)\big)-P(t)\xi_0'' \qquad (9)$$

The corresponding boundary conditions (5) for $\xi(t,x),\quad x\in[0,1]$ are

$$x=0:\xi=0,\xi'=0;\ x=1:\xi=0,\xi'=0;\ x=a:\xi+\xi_0=0 \qquad (10)$$

The relation (6) for the axial displacement will be

$$\eta(t)=-\frac{\varepsilon}{2}\int_0^1\left[\frac{\partial\xi(t,x)+\partial\xi_0(x)}{\partial x}\right]^2 dx \qquad (11)$$

The relation for forces (3) in the dimensionless form is

$$H_a(t)=-\varepsilon R_a(t)\left.\frac{\partial\left[\xi(t,x)+\xi_0(x)\right]}{\partial x}\right|_{x=a} \qquad (12)$$

If we consider the external damping and introduce the cubic nonlinearity the eq. (9) will take form

$$\ddot\xi+2\zeta_e\dot\xi+\xi''''+P(t)\xi''+\alpha\xi^3$$
$$=\frac{R_a}{\varepsilon}\delta\big(x-a(t)\big)-P(t)\xi_0'' \qquad (13)$$

where ζ_e is the dimensionless damping factor, α is the dimensionless coefficient characterizing the nonlinearity of beam elastic forces.

The boundary problem {(9), (10)} can be reduced to the integral Fredholm equation of the second order if consider the auxiliary boundary problem [6]

$$y(x)''''=f(x),\quad x=0:\quad y=y'=0; x=1:\quad y=y'=0 \qquad (14)$$

which solution can be presented in the form of the integral equation

$$y(x)=\int_0^1 G(x,\tilde x)f(\tilde x)d\tilde x\quad\Rightarrow$$
$$G(x,\tilde x)=H(x-\tilde x)\frac{(x-\tilde x)^3}{3!}+\left[2(1-\tilde x)^3-3(1-\tilde x)^2\right]\frac{x^3}{3!}+ \qquad (15)$$
$$\left[(1-\tilde x)^2-(1-\tilde x)^3\right]\frac{x^2}{2!}$$

here $H(x-\tilde x)$ is the Heavyside function and $G(x,\tilde x)$ is the Green function of the problem (14). Thus the problem {(13), (10)} reduces to the integral equation

$$\xi(t,x)=-\int_0^1 G(x,y)f(t,y)dy-$$
$$P(t)\int_0^1\frac{\partial^2 G(x,y)}{\partial y^2}w(t,y)dy+\frac{G(x,a)}{\varepsilon}R_a \qquad (16)$$
$$f(t,y)=\ddot\xi(t,y)+2\zeta_e\dot\xi(t,y)+\alpha\xi^3(t,y),$$
$$w(t,y)=\xi(t,y)+\xi_0(y)$$

The reaction R_a is derived from the condition of zero beam deflection at the support

$$\xi(t,a)+\xi_0(a)=0\quad\Rightarrow$$
$$-\int_0^1 G(a,y)f(t,y)dy-P(t)\int_0^1\frac{\partial^2 G(a,y)}{\partial y^2}w(t,y)dy$$
$$+\frac{G(a,a)}{\varepsilon}R_a+\xi_0(a)=0\quad\Rightarrow \qquad (17)$$
$$R_a=\frac{\varepsilon}{G(a,a)}\left[\begin{array}{l}-\xi_0(a)+\int_0^1 G(a,y)f(t,y)dy+\\ P(t)\int_0^1\frac{\partial^2 G(a,y)}{\partial y^2}w(t,y)dy\end{array}\right]$$

The boundary problem {(13), (10)} is equivalent to the eqs. {(16), (17)}. Eliminating the support reaction in the eq. (16) we can present main system equation as

$$\xi(t,x)=-\int_0^1 M(x,y;a)f(t,y)dy-$$
$$P(t)\int^1 D(x,y;a)w(t,y)dy-S(x,a)\xi_0(a) \qquad (18)$$

The following notations were used

$$M(x,y;a)=G(x,y)-S(x,a)G(a,y),$$
$$S(x,a)=\frac{G(x,a)}{G(a,a)} \qquad (19)$$
$$D(x,y;a)=\frac{\partial^2 G(x,y)}{\partial y^2}-S(x,a)\frac{\partial^2 G(a,y)}{\partial y^2}$$

The integrals in the right side of the equation are calculated approximately by dividing the interval $I_x = \{x : x \in [0,1]\}$ by n equal sections with the node values $x_j = j/n, \quad j = \overline{0,n}$. We apply the following notation for the generalized coordinates – node point displacement

$$q_j(t) = \xi(t, x_j), \quad j = \overline{0,n}; \quad q_0 \equiv 0, \quad q_n \equiv 0 \tag{20}$$

The integrals in the eq. (18) with account of the boundary conditions are calculated by the method of trapezoids

$$\begin{gathered}\xi(t,x) = -\frac{1}{n}\sum_{j=1}^{n-1} M(x, y_j; a) F_j(t) - \\ P(t)\frac{1}{n}\sum_{j=1}^{n-1} D(x, y_j; a) W_j(t) - S(x,a)\xi_0(a) \\ F_j(t) = \ddot{q}_j(t) + 2\zeta_e \dot{q}_j(t) + \alpha q_j^3(t), \\ W_j(t) = q_j(t) + \xi_0(y_j)\end{gathered} \tag{21}$$

If $q_j(t)$ are known the eq. (21) allows the displacements $\xi(t,x)$ field derivation. If consider the notations (20), the eq. (21) permits the obtaining of full system for variables $q_i(t), \quad i = \overline{1, n-1}$ derivation:

$$\begin{gathered}q_i(t) = -\frac{1}{n}\sum_{j=1}^{n-1}\left[M(x_i, y_j; a) F_j(t) - P(t) D(x_i, y_j; a) W_j(t)\right] \\ -S(x_i, a)\xi_0(a)\end{gathered} \tag{22}$$

The eqs. (22) include the varying in time coefficients which are dependent on the intermediate support position $a(t)$. The position of the beam right end is defined as

$$\eta(t) = -\frac{\varepsilon}{2N}\sum_{j=1}^{N-1}\left[\frac{\partial\xi(t, x_j) + \partial\xi_0(x_j)}{\partial x}\right]^2 \tag{23}$$

where N is the number of the beam length separation. It should be noted that $N > n$.

In case when one of the net node coincides with the support position x_j the matrix of the coefficients $M(x_i, y_j; a)$ immediately degenerates, that sufficiently complicates the calculations: in physical sense it means that at the moment when the support moves under the corresponding node one of the degrees of freedom “switches off”. We suppose that the intermediate support has small but finite pliability $\varepsilon^{-1} G_a$ and thus we avoid such degeneration. The boundary condition at the moving support (10) can be written for this case as

$$x = a : \xi + \xi_0 = -G_a \frac{R_a}{\varepsilon}, G_a = \gamma G(1/2, 1/2), \quad 0 < \gamma \ll 1 \tag{24}$$

here γ is the dimensionless pliability of the support and thus the support reaction can be written in the analogical form as (17) by changing $G(a,a)$ to $\tilde{G}(a,a)$ which is determined as follows

$$R_a = \frac{\varepsilon}{\tilde{G}(a,a)}\left[\begin{gathered}-\xi_0(a) + \int_0^1 G(a,y) f(t,y) dy + \\ P(t)\int_0^1 \frac{\partial^2 G(a,y)}{\partial y^2} X(t,y) dy\end{gathered}\right] \tag{25}$$

$$\tilde{G}(a,a) = G(a,a) + \gamma G(1/2, 1/2) = (1+\tilde{\gamma}) G(a,a),$$
$$\tilde{\gamma} = \gamma \frac{G(1/2,1/2)}{G(a,a)}$$

Respectively, the displacement field (21) will take the same form by changing $G(a,a) \leftarrow \tilde{G}(a,a)$ and $M \leftarrow \tilde{M}, D \leftarrow \tilde{D}, S \leftarrow \tilde{S}$:

$$\begin{gathered}\tilde{M}(x,y;a) = G(x,y) - \tilde{S}(x,a) G(a,y), \tilde{S}(x,a) = \frac{G(x,a)}{\tilde{G}(a,a)} \\ \tilde{D}(x,y;a) = \frac{\partial^2 G(x,y)}{\partial y^2} - \tilde{S}(x,a)\frac{\partial^2 G(a,y)}{\partial y^2}\end{gathered} \tag{26}$$

Let’s introduce the following matrix notation

$$\begin{gathered}\mathbf{q}(t) = \{q_1(t), ..., q_m(t)\}^T, \quad m = n-1 \\ \mathbf{S}(t) = \{\tilde{S}(x_1, a(t)), ..., \tilde{S}(x_m, a(t))\}^T, \\ \mathbf{M} = \frac{1}{n}\begin{bmatrix}\tilde{M}(x_1, y_1; a) & \cdots & \tilde{M}(x_1, y_m; a) \\ \vdots & \ddots & \vdots \\ \tilde{M}(x_m, y_1; a) & \cdots & \tilde{M}(x_m, y_m; a)\end{bmatrix}, \\ \mathbf{D} = \frac{1}{n}\begin{bmatrix}\tilde{D}(x_1, y_1; a) & \cdots & \tilde{D}(x_1, y_m; a) \\ \vdots & \ddots & \vdots \\ \tilde{D}(x_m, y_1; a) & \cdots & \tilde{D}(x_m, y_m; a)\end{bmatrix}\end{gathered} \tag{27}$$

The eqs. (22) will arrive at

$$\begin{gathered}\mathbf{M}(t)\left[\ddot{\mathbf{q}}(t) + 2\zeta_e \dot{\mathbf{q}}(t) + \alpha\mathbf{g}\right] + \left[\mathbf{I} + P(t)\mathbf{D}(t)\right]\mathbf{q}(t) = \\ = -P(t)\mathbf{D}(t)\mathbf{q}_0 - \xi_0(a(t))\mathbf{S}(t), \quad \mathbf{g} = \{q_1^3, q_2^3, ..., q_m^3\}^T\end{gathered} \tag{28}$$

where $\mathbf{q}_0 = \{\xi_0(x_1), ..., \xi_0(x_m)\}^T$ and $\mathbf{I}$ is identity matrix of measure $m \times m$.

2.3 The equilibrium position stability under constant compressive load $P(t) = P_0 = const$ and in case of moving support

The linear system with periodic coefficients is considered. It is derived from eq. (28) and takes form:

$$\mathbf{M}(t)\left[\ddot{\mathbf{q}}(t) + 2\zeta_e \dot{\mathbf{q}}(t)\right] + \left[\mathbf{I} + P_0\mathbf{D}(t)\right]\mathbf{q}(t) = \mathbf{0}, \tag{29}$$

The position of the intermediate support is varying by the monoharmonic law

$$a(t) = a_0 + A\sin\nu t \tag{30}$$

Matrices $\mathbf{M}(t), \mathbf{D}(t)$ have the common period that is equal to the period of the support excitation $T_\nu = 2\pi/\nu$.

The determined results for the first region of the main parametric resonance for the system with $m = 5$ degrees of freedom (the beam is separated by $n = 6$ sections of equal length) in Fig.3 is presented. The system parameter values on figure are specified. The notations In Fig.3 are: γ is the pliability coefficient of the intermediate support; a_0 is the average position of the intermediate support; P_0 is the magnitude of the compressive force; m is the number of degrees of freedom; ζ_e is the external damping factor; ν and A are the frequency and amplitude of the support position vibration.

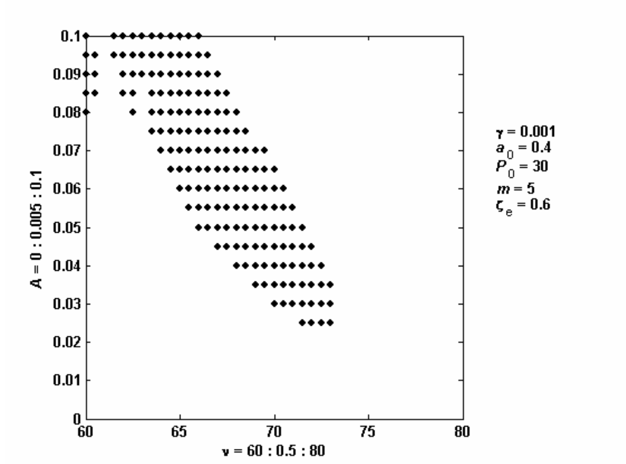

Figure 3: The region of beam dynamic instability is shown by dots:
The boundary of the instability region corresponds to the transition of the maximal multiplier through the unit circle $\mu = -1$.

3 FORCED VIBRATION MODELING

In case of the forced vibration analysis the eqs. (28) are numerically integrated. As example the vibrations under constant compressive load $P(t) = P_0 = const$ are considered. The support position variation are given in form(30). The calculated results for two cases are presented: the system is out of the instability region (1) and entire of the instability region (2). The specified parameters are follows:

$$\alpha = 10, \gamma = 0.001, P_0 = 30, \zeta_e = 0.9, m = 5,$$
$$\varepsilon = 0.01, a_0 = 0.4;\ case(1):\quad \nu = 60, A_0 = 0.05; \qquad (31)$$
$$case(2):\quad \nu = 73, A_0 = 0.025$$

The initial imperfects $\left\|\xi_0(x)\right\| \neq 0$ in the first case are taken as

$$q_{0j} = \frac{1}{N_G} G(x_j, x_j), \quad N_G = \max_j \left| G(x_j, x_j) \right|, \quad j = \overline{1, m} \qquad (32)$$

and for the second case, regarding that we are into the dynamic instability region of beam straight position, we can consider the dynamics of ideal beam.

The results of the numeric modeling for the case of vibration excitation out of the instability region in Figures 4 and 5 are presented. As it is seen from figures the lateral beam deflection and the axial displacement of tool right end and the reaction of moving support are different if vary the support excitation frequency. The amplitudes of the vibrations are small and application of such excitation mechanism is not effective.

The analogical results for the case of the vibration excitation in the region of the first main parametrical resonance in Fig. 6 and Fig. 7 are presented. We can note that the tool lateral vibrations have the frequency which is equal to the half of the moving support vibration frequency (vibration period is equal to $2T_\nu$), and the right support vibrates in axial direction with the same frequency as the support. The dimensionless amplitude of vibrations $\eta(\tau)$ has the magnitude of order 1 which is sufficient for the discontinuous cutting realization.

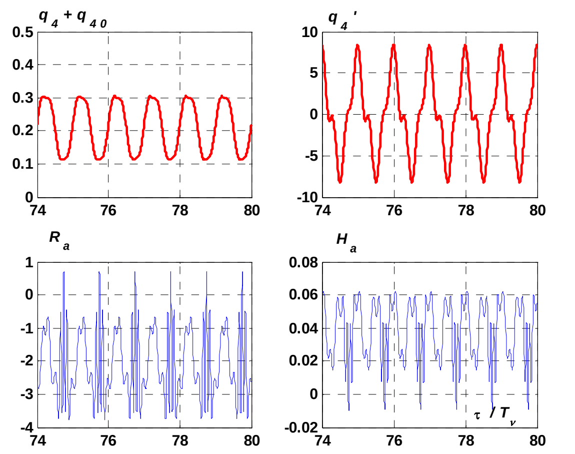

Figure 4: Evolution of generalized coordinate and velocity $q_4(\tau), q_4'(\tau)$, lateral and axial reaction of moving support $R_a(\tau), H_a(\tau)$.

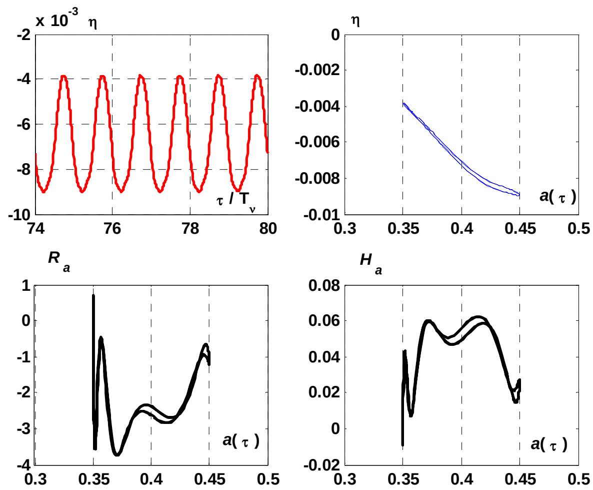

Figure 5: Evolution of the beam right end displacement $\eta(\tau)$, and variation of lateral and axial reaction of moving support $R_a(\tau), H_a(\tau)$ depending on the support position.

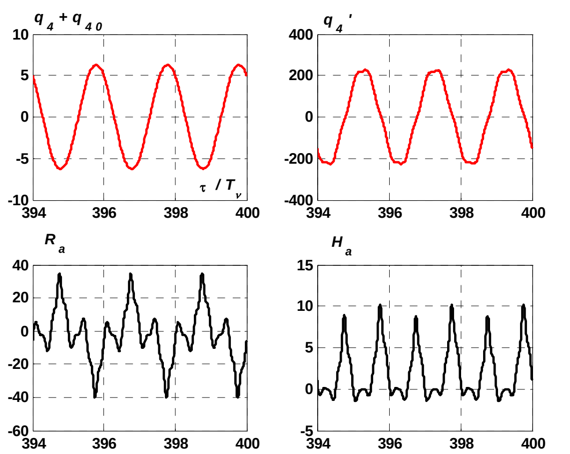

Figure 6: Evolution of generalized coordinate and velocity $q_4(\tau), q_4'(\tau)$, lateral and axial reaction of moving support $R_a(\tau), H_a(\tau)$.

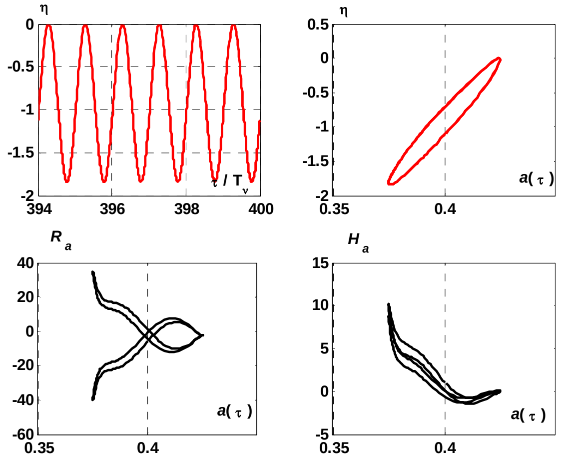

Figure 7: Evolution of the beam right end displacement $\eta(\tau)$, and variation of lateral and axial reaction of moving support $R_a(\tau), H_a(\tau)$ depending on the support position.

Therefore the considered scheme of the vibratory drilling realization occurs to be efficient if the parameters correspond to the operation in the region of the main parametric resonance of the tool lateral vibrations. Particularly, if the frequency of tool rotation is equal to ν_C then the support vibration frequency should be specified as [4, 5]: $\nu \approx 1.7\nu_C, \nu \approx 3.4\nu_C$.

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